



# Comparing Blade/Soil Interaction Models in a Matlab Program to Measurements of Forces to Push Narrow Rods Through Sand and Simulant Materials for Design of Extraterrestrial Soil Handling Machines

R. H. King<sup>1</sup>, P. J. van Susante<sup>2</sup>, and M. A. Gefreh<sup>3</sup>

<sup>1</sup>EG Division, CSM, 279 Brown Hall, Golden, CO 80401, [rking@mines.edu](mailto:rking@mines.edu),

<sup>2</sup>EG Division, CSM, Golden, CO 80401,

<sup>3</sup>Sierra Nevada Corporation Space Systems, 8130 Shaffer Parkway, Littleton, CO 80127



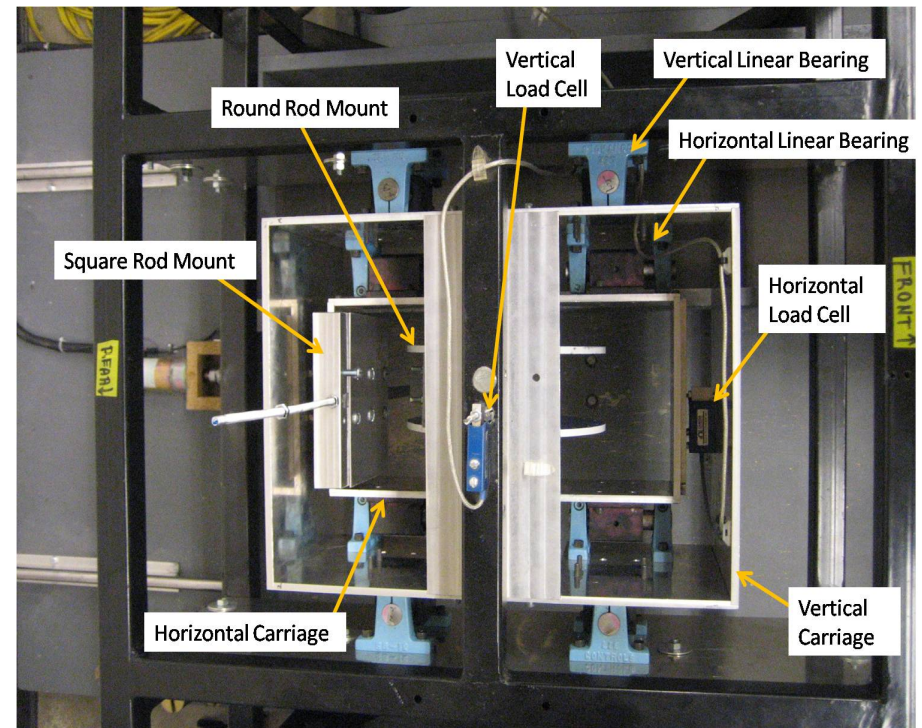
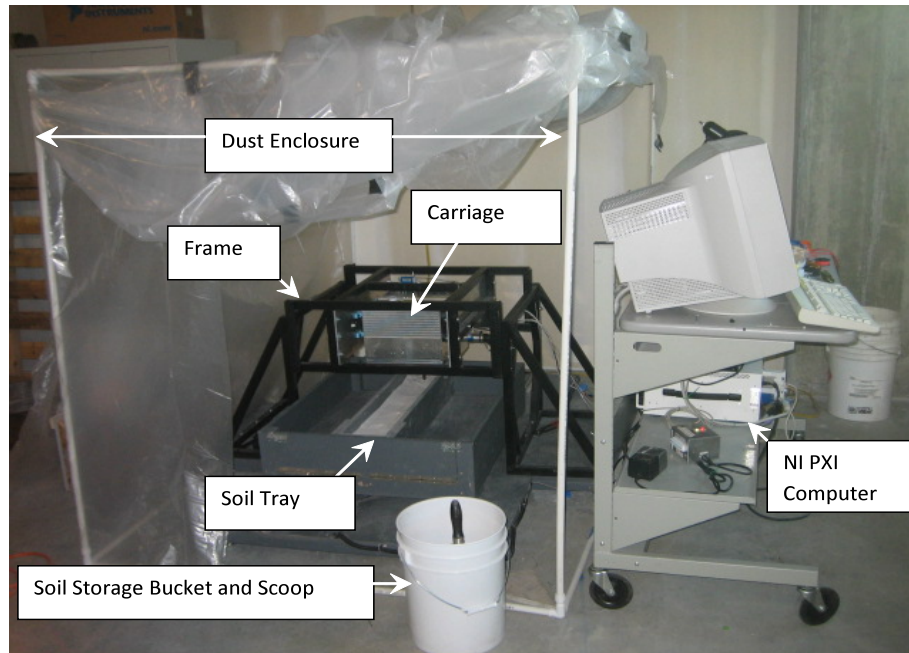
## Goals



- Develop a correlation between analytical excavation models and experimental data with lunar simulants



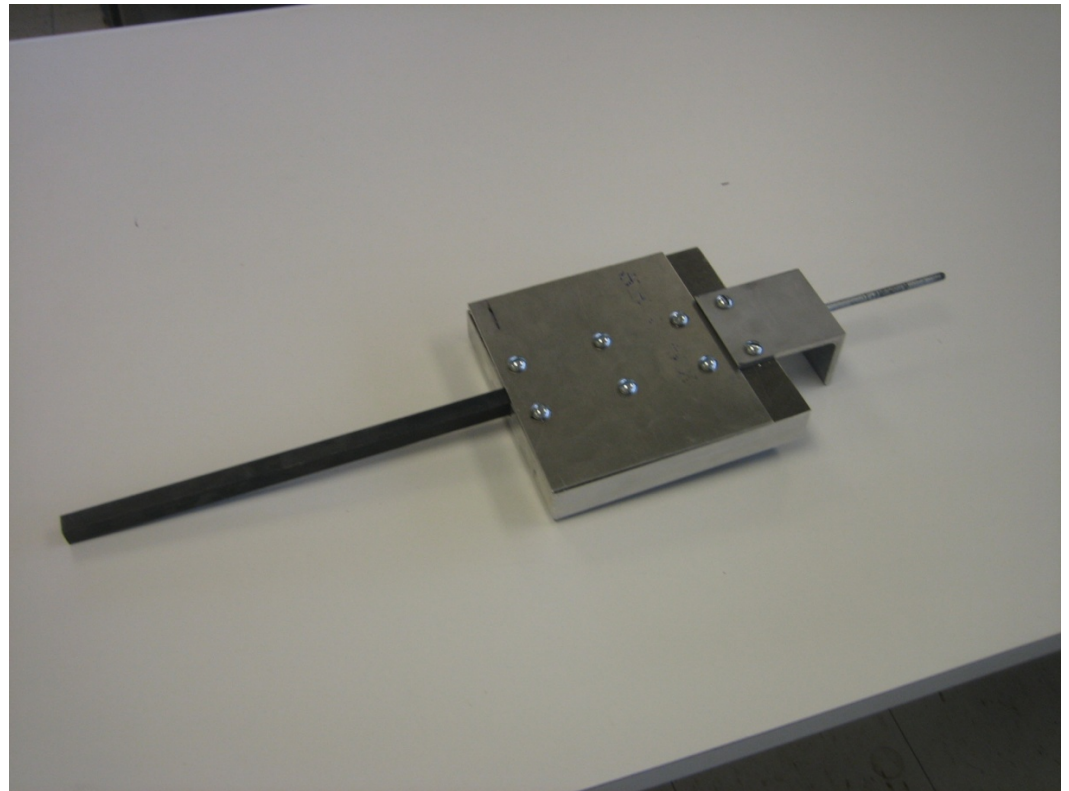
# The Testing Apparatus



- Fittings for  $\frac{1}{2}$  inch diameter rod and  $\frac{1}{2}$  in square rod.
- Raise/lower rod into material
- Material translated horizontally
- Horizontal and Vertical forces decoupled through linear bearings
- 111N capacity load cell measured horizontal force with full scale accuracy of 1.3%



# Cutting Tools





## Soils



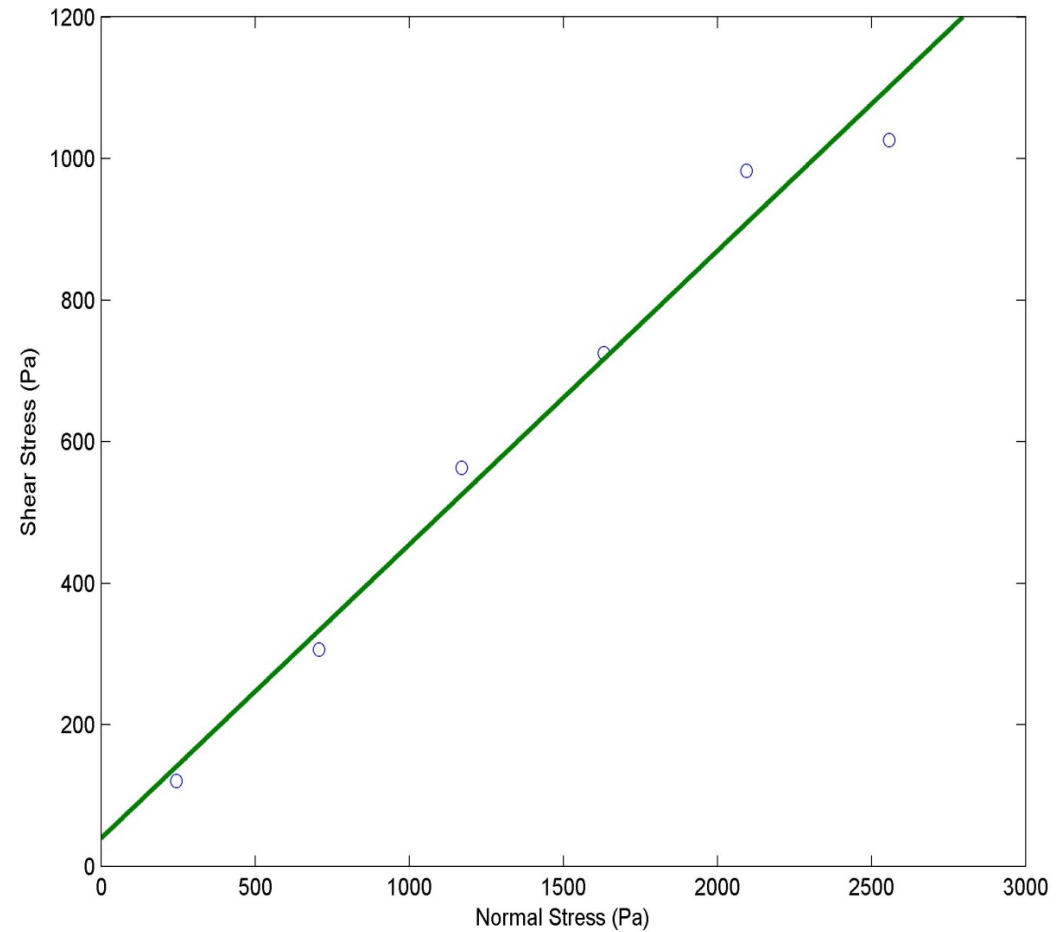
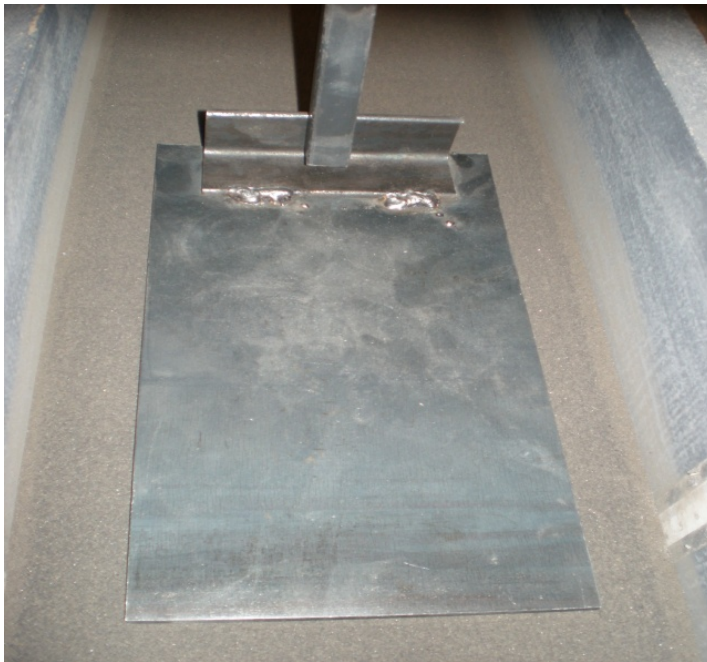
- Ottawa Sand (F-110 from US Silica)
  - Pure Silica
  - Uniform Grain Size
  - Rounded to sub-angular grains
- JSC-1A
  - Based on lunar soil sample 14163 (Apollo 14)
  - Milled and sieved, glass-rich, basaltic material
  - Very Angular
  - No chemical alteration
  - No Agglutinates

Sieve Size	Minimum Particle Size (µm)	% Retained on Individual Sieve		Cumulative % Retained on Sieve	
		Ottawa	JSC-1A	Ottawa	JSC-1A
20	841	0.0	0.0	0.0	0.0
30	595	0.0	7.5	0.0	7.5
40	420	0.0	7.5	0.0	15.0
50	297	1.2	7.5	1.2	22.5
70	210	8.3	6.3	9.5	28.8
100	149	15.7	13.2	25.2	42.0
140	105	43.9	6.8	69.1	48.8
200	74	25.2	10.0	94.3	58.8
270	53	5.7	10.7	100.0	69.5
Pan	-	0.0	30.5	100.0	100.0



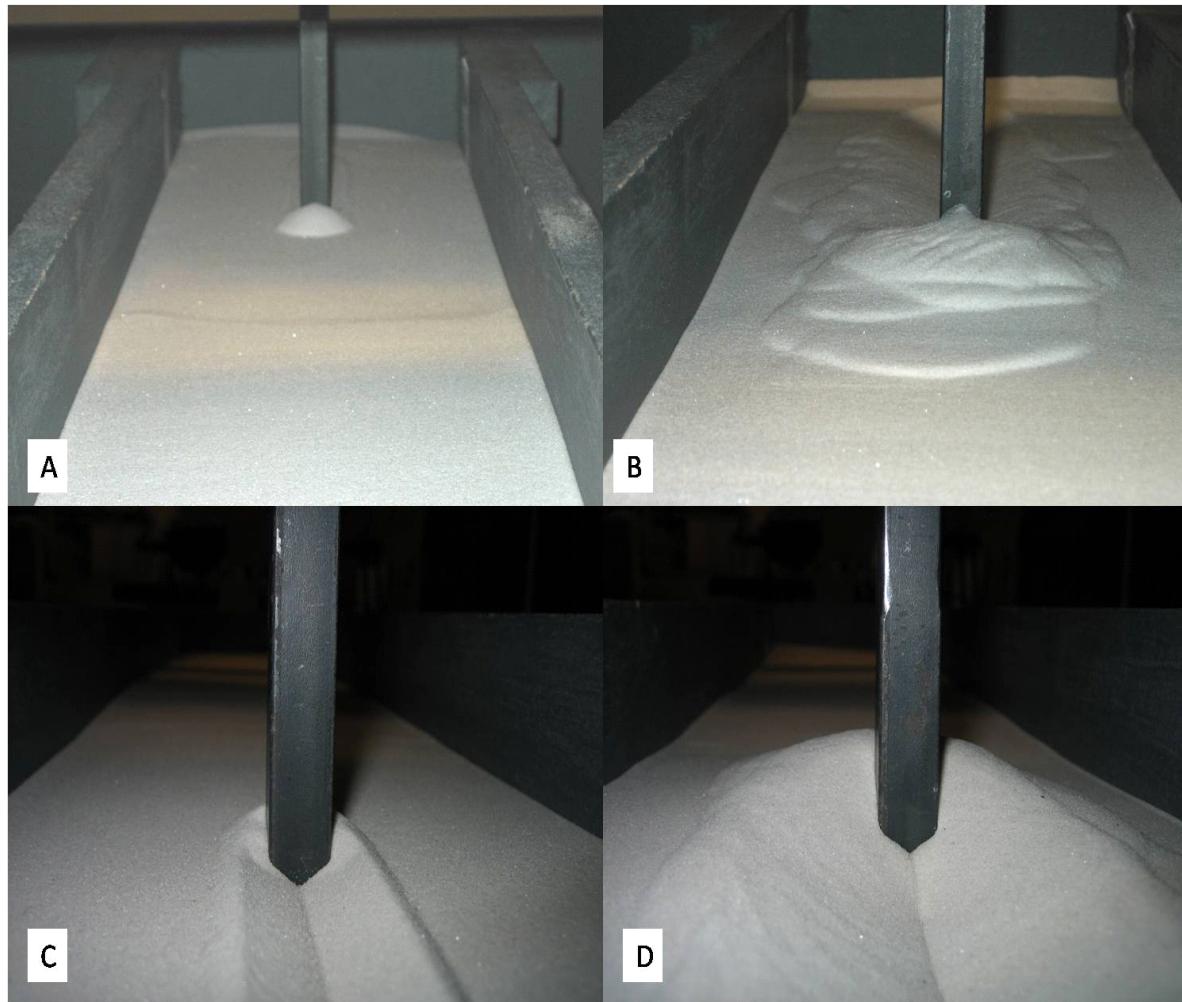


# Measurement of External Angle of Friction and Adhesion





## Pictures of Soil Cuts



Square rod pushing through Ottawa sand (A) The front view at a cut depth of 0.635 cm. The surcharge height is about 0.6 cm (B) The front view at a cut depth of 6.985 cm. The surcharge height is about 1.0 cm. (C) The rear view at a cut depth of 0.635 cm. (D) The rear view at a cut depth of 6.985 cm.





## Pictures of Soil Cuts

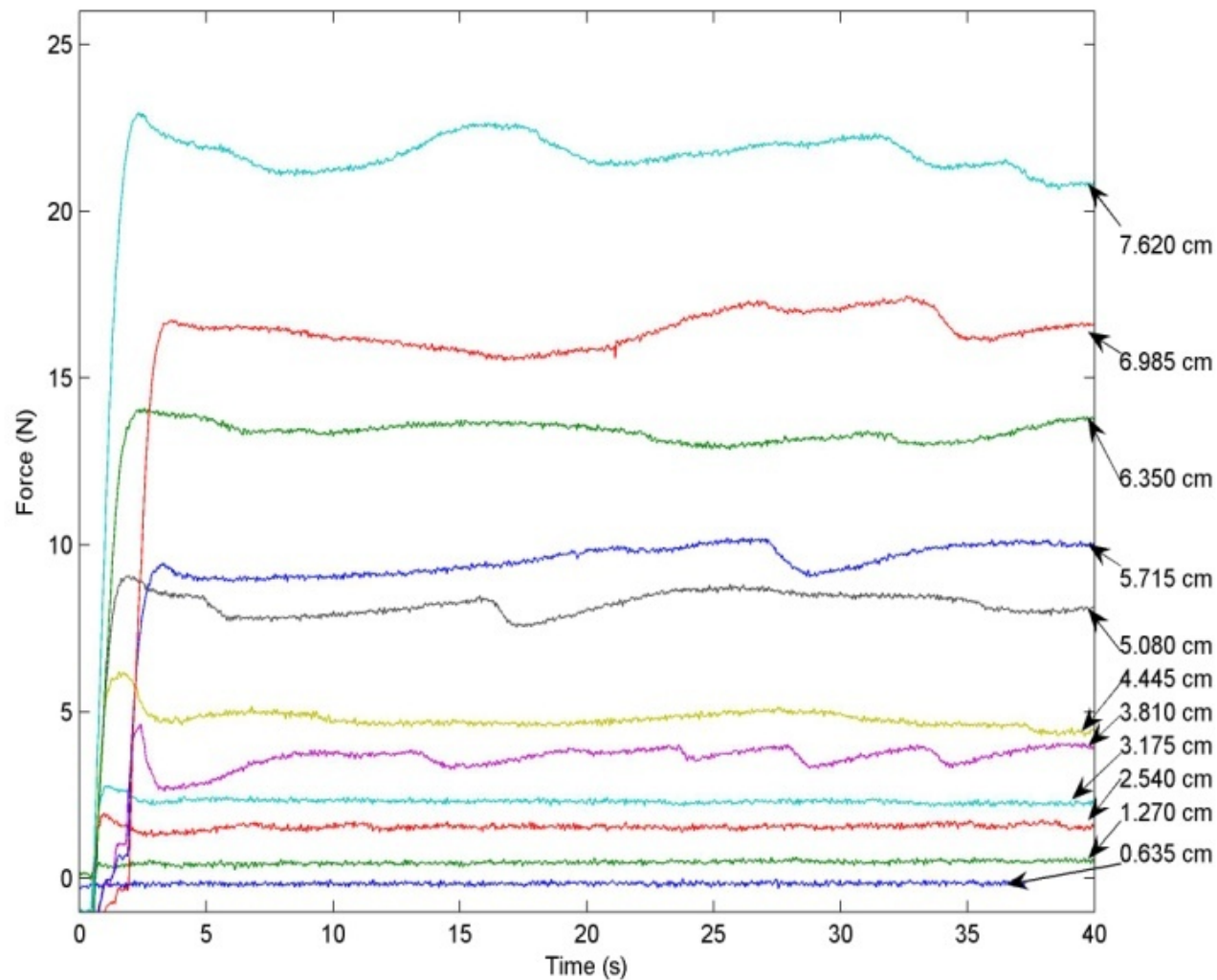


Square rod pushing through JSC-1A lunar simulant (A) The back view at a cut depth of 0.635 cm. (B) The front view at a cut depth of 6.985 cm. The surcharge is about 1.25 cm. (C) The back view at a cut depth of 0.635 cm. (D) The back view at a cut depth of 6.985 cm.



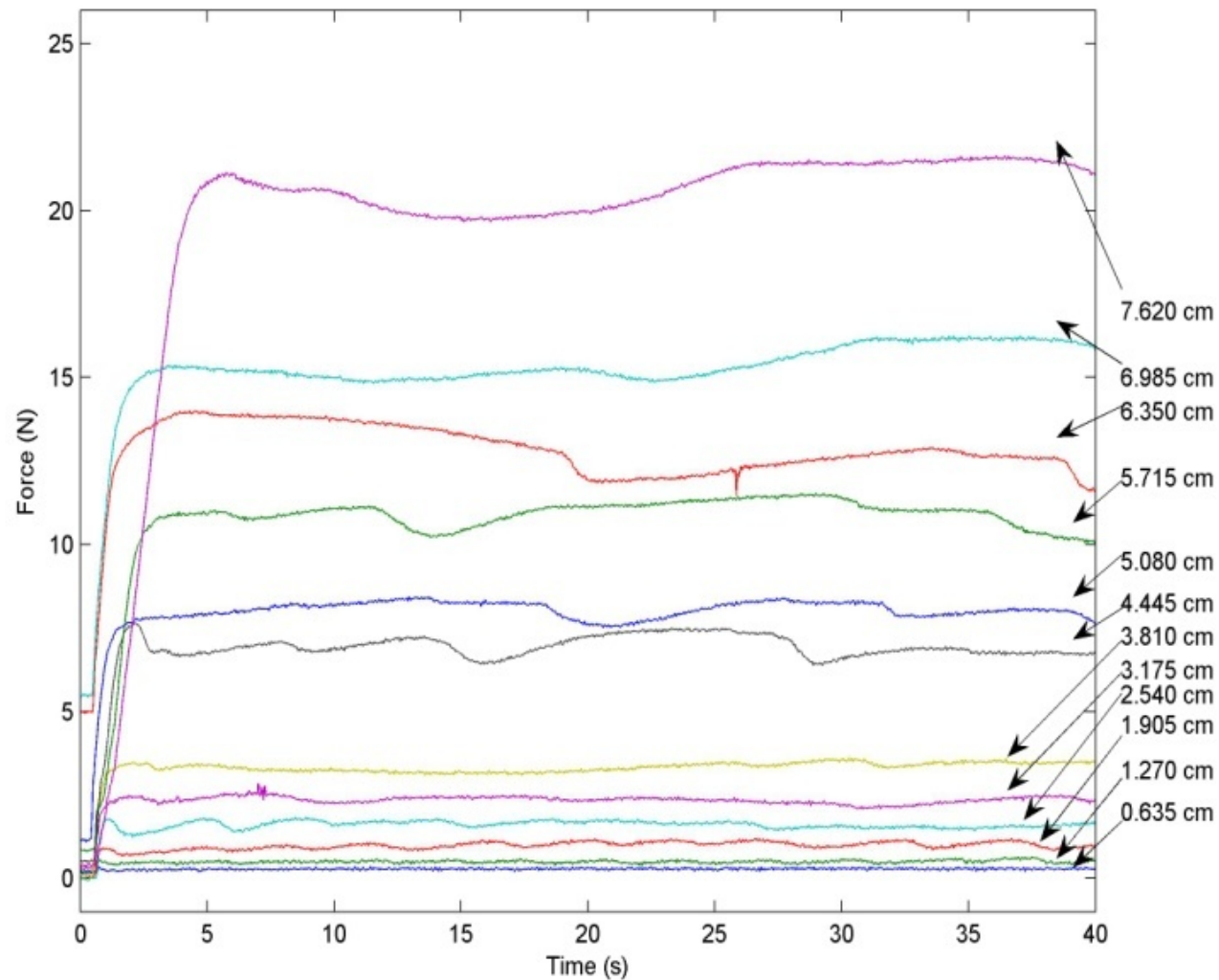


# Cut Measurements for Round Rod in Ottawa Sand



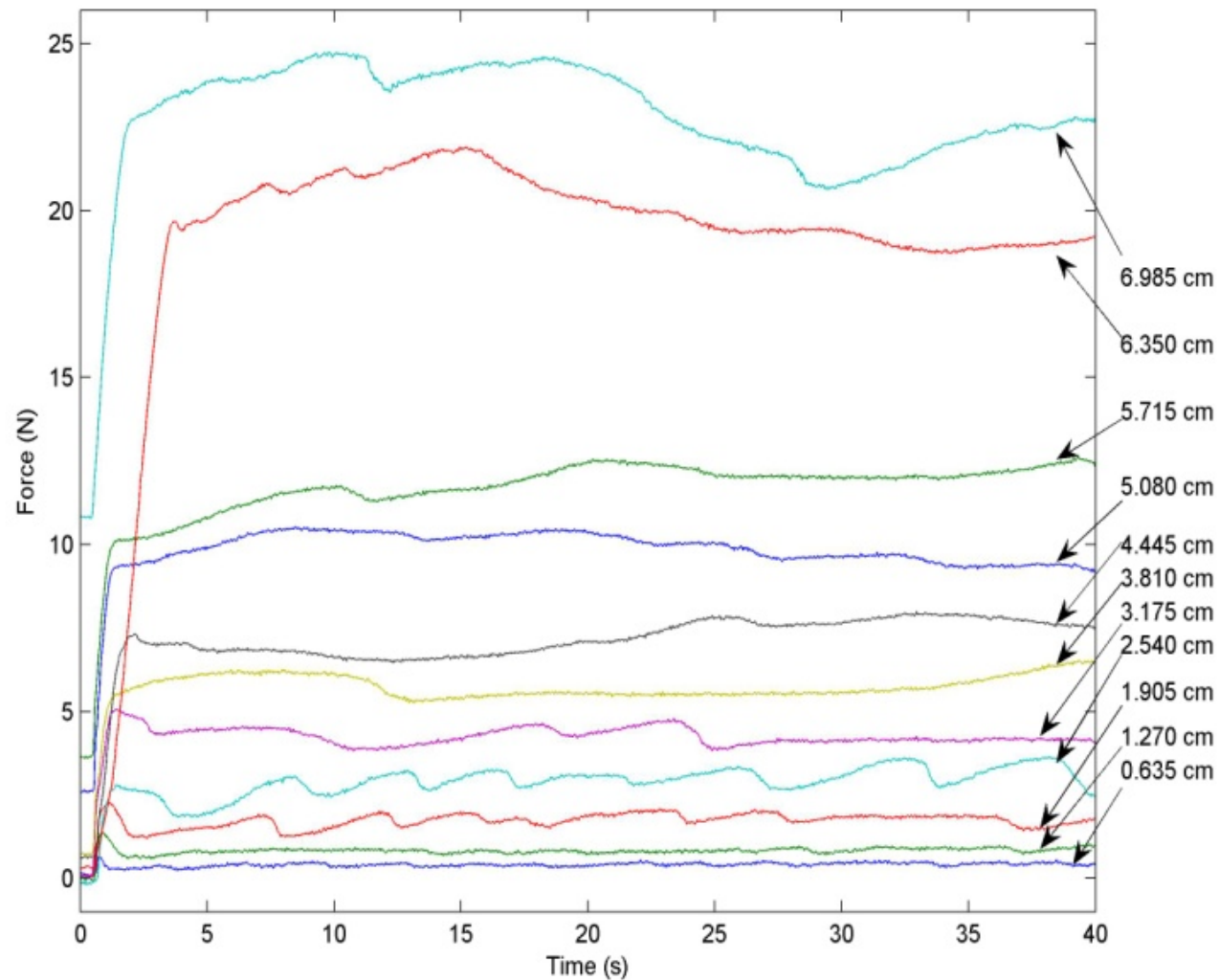


# Cut Measurements for Square Rod in Ottawa Sand





# Cut Measurements for Square Rod in JSC-1A



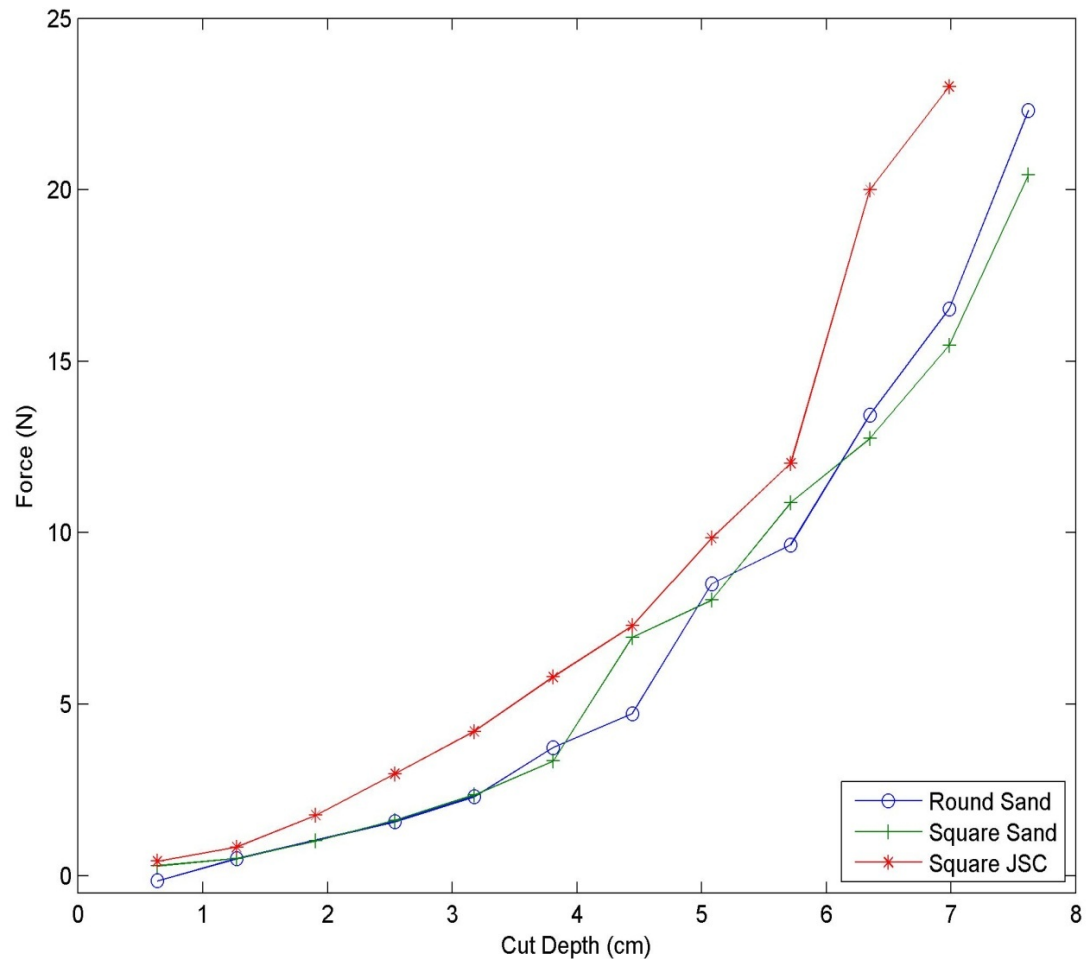




# Force Comparison

## Results:

- Shape Comparison
  - Little difference
- Material Comparison
  - JSC-1A requires more force
- Boundary Effects
  - JSC-1A far more sensitive
- System limitations
  - Minimum measurable force of 0.2 N





# Analytic Models

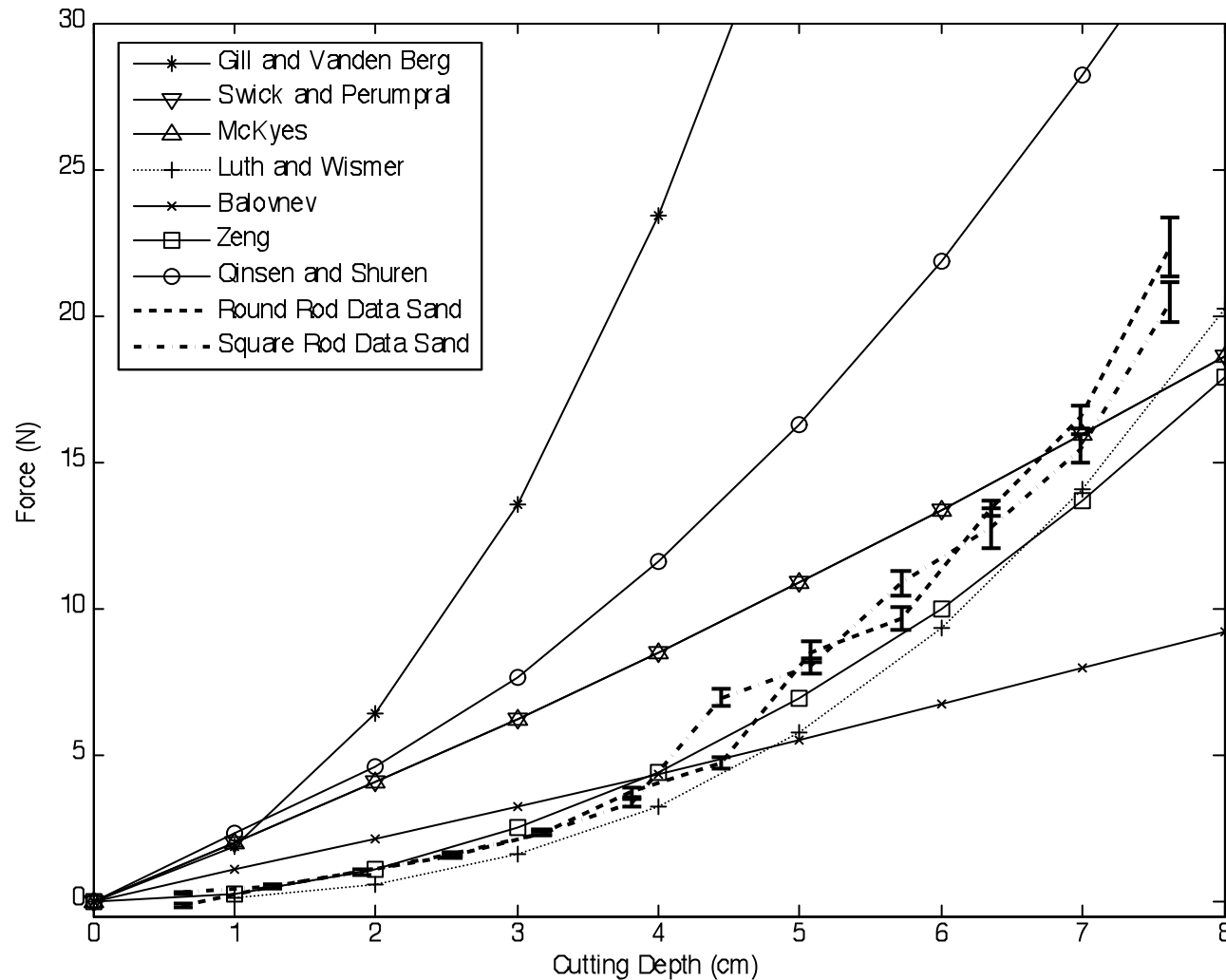


- Models chosen for 2D fail pattern and low cohesion materials
- Included Models
  - Balovnev
    - Experimentally compared to sand, sandy loam, and loam soils
  - Gill and Vanden Berg
    - Comprehensive handbook with clay and sand
  - Luth and Wismer
    - Emperically developed from sand measurements
  - McKeys
  - Osman
    - Compared to dry sand, wet sand, and clay measurements
  - Qinsen and Shuren
  - Swick and Perumpral
  - Zeng
    - Developed from fundamental soil mechanics

Description	Symbol		JSC-1A	Unit
coefficient of passive earth pressure	$K_0$	0.573	0.573	
cohesion	$c$	1.4	1.7	kPa
cohesion after cutting	$c_0$	0.7	1.0	kPa
cutting depth	$d$	0.0-0.08	0.0-0.08	m
earth acceleration of gravity	$g$	9.81	9.81	m/s <sup>2</sup>
external angle of friction	$\delta$	17	23	°
Gill's cut resistance	$K$	1	1	N/m
horizontal acceleration	$a_h$	0	0	m/s <sup>2</sup>
internal friction angle	$\phi$	30	37	°
Internal friction angle after cutting	$\phi_0$	30	37	°
shear plane failure angle	$\rho$	35	29	°
soil bulk density	$\gamma$	1.0	1.7	g/cm <sup>3</sup>
soil bulk density after cutting	$\gamma_1$	0.7	1.4	g/cm <sup>3</sup>
soil-tool adhesion	$Ca$	39	39	Pa
Soil-tool normal force	$N_0$	1000	1000	N
surcharge pressure	$q$	25.4	43.18	kg/m <sup>2</sup>
tool angle of curvature	$\theta$	0.001	0.001	°
tool edge angle	$\alpha$	0	0	°
tool edge thickness	$eb$	0	0	m
tool height	$H$	$d + 0.1$	$d+0.125$	m
tool horizontal acceleration	$a_h$	0	0	m/s <sup>2</sup>
tool length	$L$	H	H	m
tool mass	$Wb$	0.4	0.4	kg
tool radius of curvature	$R$	10000	10000	m
tool rake angle	$\beta$	89	89	°
tool side length	$L_s$	H	H	m
tool side width	$s$	w	w	m
tool velocity	$v$	0.0033	0.0033	m/s
tool width	$w$	0.0127	0.0127	m
tool vertical acceleration	$a_v$	0	0	m/s <sup>2</sup>



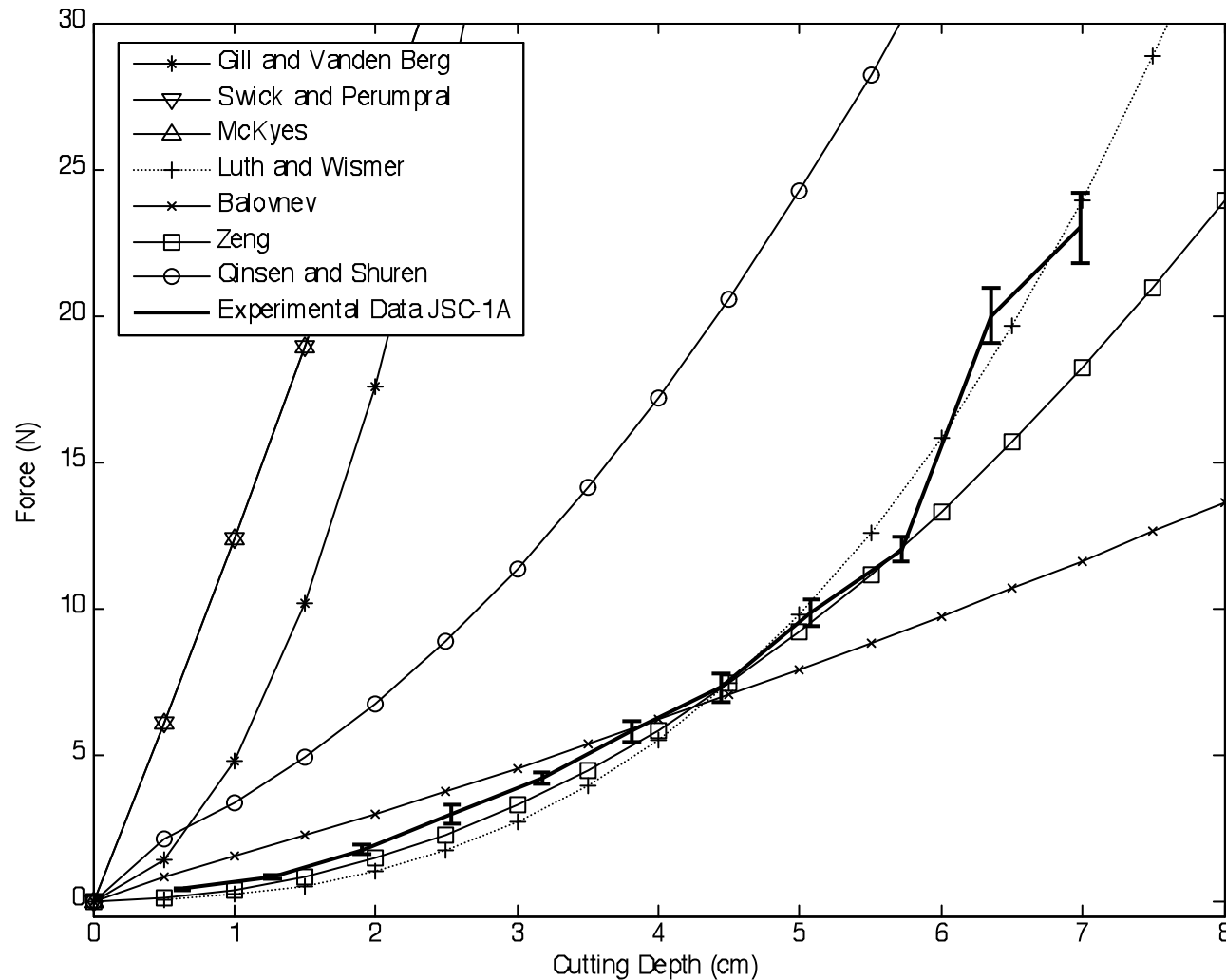
# Comparison between Measured forces and Analytic Models







# Comparison between Measured forces and Analytic Models





# Conclusions



- Ottawa Sand
  - The Luth and Wismer model and the Zeng model showed the closest fit over the measurements.
  - Balovnev, McKeys, and Swick and Perumpral all diverge due to linearity
  - The Gill and Vanden Berg model and the Qinsen and Shuren model show equivalent curves but significantly over estimate the forces
  - Osman was unstable over this range
- JSC-1A
  - The Luth and Wismer model and the Zeng model showed the closest fit over the measurements.
  - Balovnev fits for smaller cut depths, but diverges due to linearity
  - McKeys, and Swick and Perumpral still linear, but significantly over estimates forces
  - The Gill and Vanden Berg model and the Qinsen and Shuren model show equivalent curves but significantly over estimate the forces
  - Osman was unstable over this range
- Depth significantly increases forces. Equipment should be designed to make shallow cuts
- Future work will expand to various shapes, blade sizes, increasing resolution, and model distributed forces with finite element software



# Backup Slides







# Test Bed Specifications



- Modified from micro-scale bucket wheel excavation force measurement apparatus
- Fittings for 1/2 inch diameter rod and 1/2 in square rod.
- Separates horizontal and vertical forces through linear rod bearings
- 111 N load cell measured horizontal forces
- 222 N Load cell measured vertical forces
- Carriage is 169 N
- NI6070E multi-functioned DAQ board
  - 16 Analog inputs
  - 2 Analog outputs
  - 8 digital I/O
  - 2 counter/timers
- NI PXI 7344
  - 4 Axis servo motor controller
- NI MID-7652
  - Controls Speed and Trajectory of Motor
  - PID control constants
    - P=91
    - I=535
    - D=2
- Pittman tray motor
  - 30.3 VDC
  - 10.0625:1 gear ratio
  - 2048 CPR resolution optical encoder
  - 0.5% speed error empty, 1% speed error when full
- Full Scale Horizontal error is 1.3%
- Full Scale Vertical error is 1.94%
- 0.0033 m/s cutting velocity
- Data recorded at 300S/s



# Balovnev Model



- wide blade
- based on the theory of the limiting equilibrium of soils
- assumes cut depth is less than width
- assumes full surcharge pile
- measurements in dry sand, sandy loam and loam. Theoretical values were 3-9% higher than the measured results

$$H_{balovnev-bulldozer} = (1 + \cot(\beta) \tan(\delta)) A_1 w d \left[ \frac{\gamma g d}{2} + c \cot(\phi) + \frac{\gamma_1 g \tan(\phi) (\cos(\phi))^2 H^2}{d K_\psi} + \gamma_1 g H \right] +$$

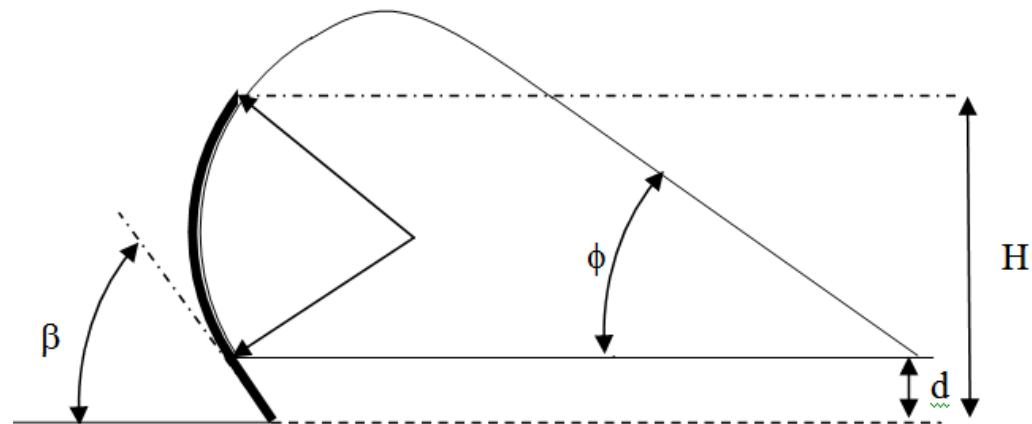
$$\frac{\gamma_1 g w H^2 (\cos(\phi))^2}{2}$$

$$\text{Where } A_1 = \frac{(1 - \sin(\phi) \cos(2\beta))}{1 - \sin(\phi)}$$

$$\text{and } K_\psi = \frac{\left( \tan(\beta) + \tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \right)}{\tan(\beta) \tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}$$

$$T_{balovnev-bulldozer} = H_{balovnev-bulldozer} \csc(\beta + \delta)$$

$$V_{balovnev-bulldozer} = H_{balovnev-bulldozer} \cot(\beta + \delta)$$





# Gill and Vanden Berg Model



- agriculture emphasis
- no surcharge
- straight failure surface
- includes terms for:
  - inertia of the soil,
  - soil-soil cohesion
  - soil mass

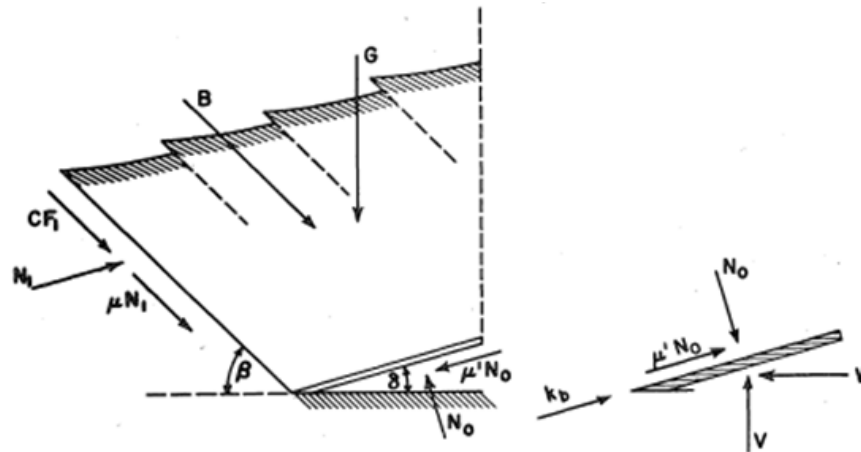
$$H_{gvdb-b} = N_0 \sin(\beta) + \delta N_\phi \cos(\beta) + Kw$$

$$H_{gvdb-b} = \left( \frac{wd(\sin(\beta) + \delta \cos(\beta))(\sin(\rho) + \phi \cos(\rho))}{\sin((\rho + \beta)(1 - \phi\delta)) + \cos((\beta + \rho)(\phi - \delta))} \right) *$$

$$\left[ \left( g\gamma \frac{\sin(\beta + \rho)}{\sin(\rho)} \right) \left( l + \left( \frac{d \cos(\beta + \rho)}{2 \sin(\rho)} \right) + \left( \frac{d \sin(\beta + \rho) \tan(\beta)}{2 \sin(\rho)} \right) \right) + \left( \frac{c}{(\sin(\rho) + (\phi \cos(\rho))) \sin(\rho)} \right) + \left( \frac{\gamma v^2 \sin(\beta)}{(\sin(\beta + \rho))(\sin(\rho) + \phi \cos(\rho))} \right) \right] + Kw$$

$$T_{gvdb-b} = H_{gvdb-b} \csc(\beta + \delta)$$

$$V_{gvdb-b} = H_{gvdb-b} \cot(\beta + \delta)$$



Blouin S, Hemami A, Lipsett M. Review of resistive force models for earthmoving processes. J Aerospace Eng 2001;14(3):102–11.

Slide 20 Gill WR, Vanden Berg GE. Soil Dynamics in Tillage and Traction. Agriculture handbook no. 316. Agricultural Research Service US Dept of Agriculture; 1968.





# Luth and Wismer Sand Model



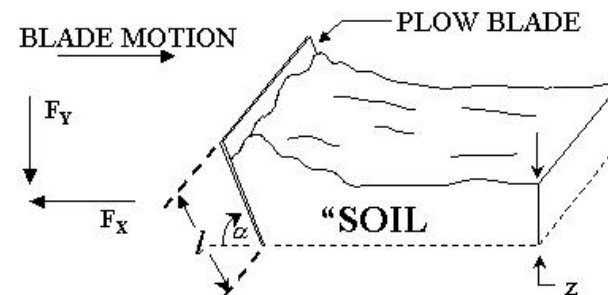
- narrow tillage tools
- based on dimensional analysis of empirical data
- no surcharge
- Separate model for clay
- measurements in sand with 30, 45, 60, 90 and 105° rake angles
- horizontal force: 48.9-N std. error, force range: 03.3 – 1334.5 N (13%)
- vertical force: 26.7-N std. error, force range: -711.7 to 556.03 N (13%)
- Used by Moore et al for Viking scoop analysis

$$H_{l\&w-sand} = \beta^{1.73} \left( \frac{d}{L \sin \alpha} \right)^{0.77} \left( 1.05 \left( \frac{d}{w} \right)^{1.1} + 1.26 \left( \frac{v^2}{gL} \right) + 3.91 \right) \gamma w d^{0.5} L^{1.5}$$

$$V_{l\&w-sand} = \left[ 1.93 - (\beta - 0.714)^2 \right] \left( \frac{d}{L \sin \beta} \right)^{0.777} \left( 1.31 \left( \frac{d}{w} \right)^{0.966} + 1.43 \left( \frac{v^2}{gL} \right) + 5.6 \right) \gamma w d^{0.5} L^{1.5}$$

$$T_{l\&w-sand} = \sqrt{(H_{l\&w-sand})^2 + (V_{l\&w-sand})^2}$$

$\rho$  = DENSITY OF "SOIL"  
 $g$  = ACCELERATION OF GRAVITY  
 $b$  = WIDTH OF BLADE  
 $l$  = HEIGHT OF BLADE  
 $z$  = OPERATING DEPTH  
 $\alpha$  = BLADE ANGLE (RADIAN)  
 $V$  = VELOCITY  
 $C$  = COHESION



Luth HJ, Wismer RD. Performance of plane soil cutting blades in sand. Trans ASAE 1971; 255–9.



# Mckyes Model

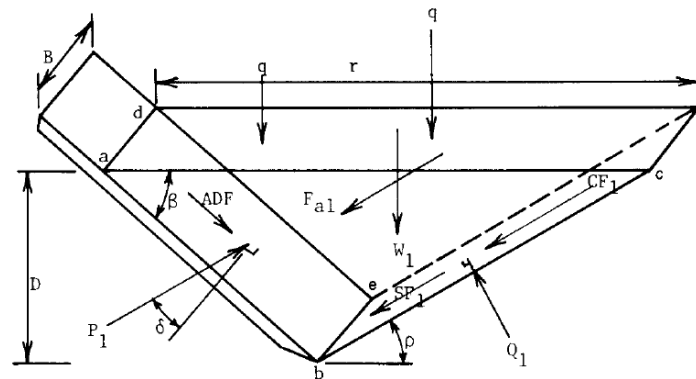


- based on Reese
- narrow tillage tools
- includes:
  - soil-tool adhesion,
  - mass,
  - inertia,
  - Surcharge
  - cohesion terms
- same results as S&P
- center and side wedges
- tested in sand and sandy loam with 1.25-25 cm blade widths, 0.25-5 width to depth ratios, from 35-63° soil failure angles, 30-90° rake angles. 14-inch blade field test matched well with predictions for rake angles of 45-60°, but 20% error at 90 °.

$$T_{mckyes} = \left( \frac{wd}{\cos(\beta + \delta) + (\sin(\beta + \delta) \cot(\rho + \phi))} \right) \left[ \left( \frac{\gamma g d (\cot(\beta) + \cot(\rho))}{2} \right) + g q (\cot(\beta) + \cot(\rho)) + c(1 + \cot(\rho) \cot(\rho + \phi)) + C_a(1 - \cot(\beta) \cot(\rho + \phi)) + \left( \frac{\gamma v^2 (\tan(\rho) + \cot(\rho + \phi))}{1 + \tan(\rho) \cot(\beta)} \right) \right]$$

$$H_{mckyes} = T_{mckyes} \sin(\beta + \delta)$$

$$V_{mckyes} = T_{mckyes} \cos(\beta + \delta)$$



Mckyes E. Soil cutting and tillage. Developments in agricultural engineering, vol. 7. Amsterdam: Elsevier; 1985.



# Osman Model

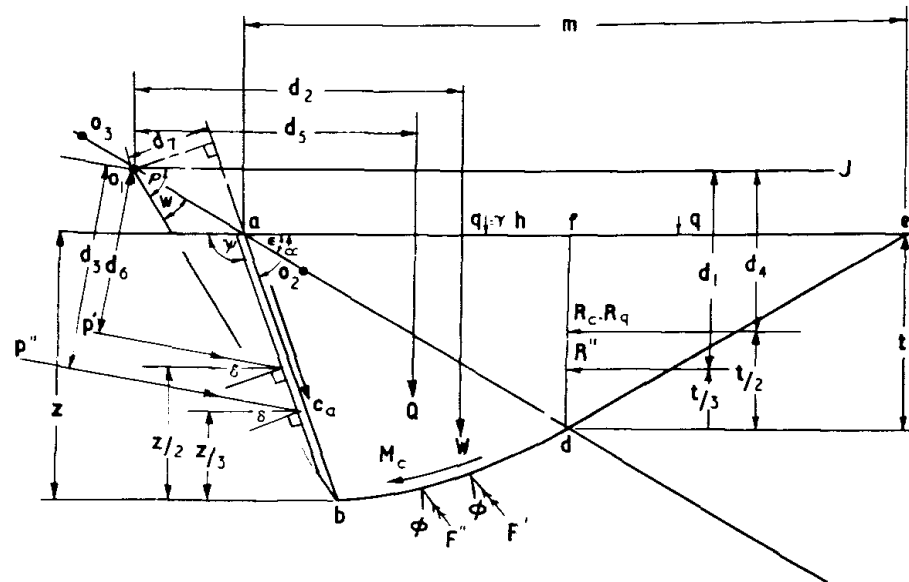
- wide blade
- surcharge is a uniform distributed pressure
- based on passive earth pressure theory
- failure surface is composed of equiangular spiral + a straight surface requiring a local minimization of  $dP/d\lambda$
- measurements on 24"x4"blades, of 30, 50, 70, 90 and 105° rake angles, in sand, clay, and mixture.

$$T_{osman} = w \left[ \frac{\left[ \left( \frac{1}{2} g \gamma t^2 (\tan(45 + \frac{\phi}{2}))^2 d_1 \right) + \left( \left( \frac{r_0^2}{4 \tan(\phi)} g \gamma (e^{2w \tan(\phi)} - 1) \right) d_2 \right) + \left( (g q t (\tan(45 + \frac{\phi}{2}))^2 d_4 \right) \right]}{d_3} \right] +$$

$$w \left[ \frac{\left[ \left( \left( \frac{c}{2 \tan(\phi)} (r_1^2 - r_0^2) \right) + \left( (2 c t \sqrt{(\tan(45 + \frac{\phi}{2}))^2} d_4 \right) + (x_d q g d_5) + (C_d l d_7) \right) \right]}{d_6} \right]$$

$$H_{osman} = T_{osman} \sin(\beta + \delta)$$

$$V_{osman} = T_{osman} \cos(\beta + \delta)$$



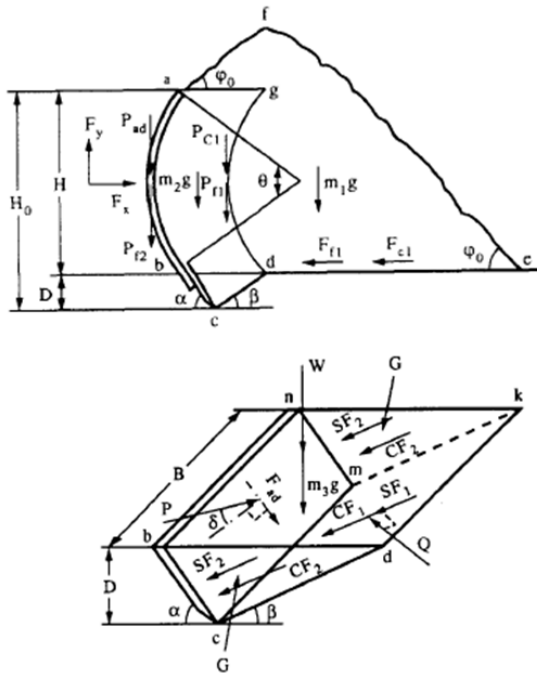
Osman M. The mechanics of soil cutting blades. J Agric Eng Res 1964; 9(4):313-28.



# Qinsen and Shuren Model



- wide blade
- Model is composed of a surcharge component and a cutting component
- measurements on 389, 468 and 600-mm blade widths, 20 to 30-mm cut depths, 105 to 170-mm blade heights, of 74 to 116-mm curvature radii, and 45° rake angle in. Sandy clay



$$P_x = 2S_{F2} \cos(\beta) + Q \sin(\beta) + S_{F1} \cos(\beta) - F_{ad} \cos(\alpha) + 2C_{F2} \cos(\beta) + C_{F1} \cos(\beta)$$

$$W = P_{f1} + P_{f2} + P_{ad} + P_{c1} + m_{2g} + m_{3g}$$

$$P_y = W + 2S_{F2} \sin(\beta) + S_{F1} \sin(\beta) + 2C_{F2} \sin(\beta) + F_{ad} \sin(\alpha) + C_{F1} \sin(\beta) - Q \cos(\beta)$$

$$F_x = P_x + F_{f1} + F_{c1}$$

$$F_y = P_y - P_{f2} - P_{ad}$$

$$F_{tot} = \sqrt{F_x^2 + F_y^2}$$

surcharge component

$$F_{c1} = c_0 w (H + 2d \tan(\phi_0)) \cot(\phi_0)$$

$$m_{1g} = 0.5 \gamma_0 w (H + 2d \tan(\phi_0))^2 \cot(\phi_0)$$

$$F_{f1} = m_{1g} \tan(\phi)$$

$$P_{f1} = (F_{f1} + F_{c1}) \tan(\phi)$$

$$P_{f2} = (F_{f1} + F_{c1}) \tan(\delta)$$

$$m_{2g} = \gamma_0 w H 2d$$

$$P_{c1} = c_0 w R \theta$$

$$P_{ad} = C_a w R \theta$$

cutting component

$$G = \frac{1}{3} \gamma d (1 - \sin(\phi)) \frac{1}{2} d^2 (\cot(\alpha) + \cot(\beta))$$

$$m_{3g} = \frac{1}{2} \gamma w d^2 (\cot(\alpha) + \cot(\beta))$$

$$C_{F1} = \frac{c w d}{\sin(\beta)}$$

$$C_{F2} = \frac{1}{2} c d^2 (\cot(\alpha) + \cot(\beta))$$

$$S_{F2} = G \tan(\phi)$$

$$S_{F1} = Q \tan(\phi)$$

$$F_{ad} = \frac{ad * w d}{\sin(\alpha)}$$

Qinsen Y Shuren S. A soil-tool interaction model for bulldozer blades. J Terramech 1994; 31(2):55-65.



# Swick and Perumpral Model

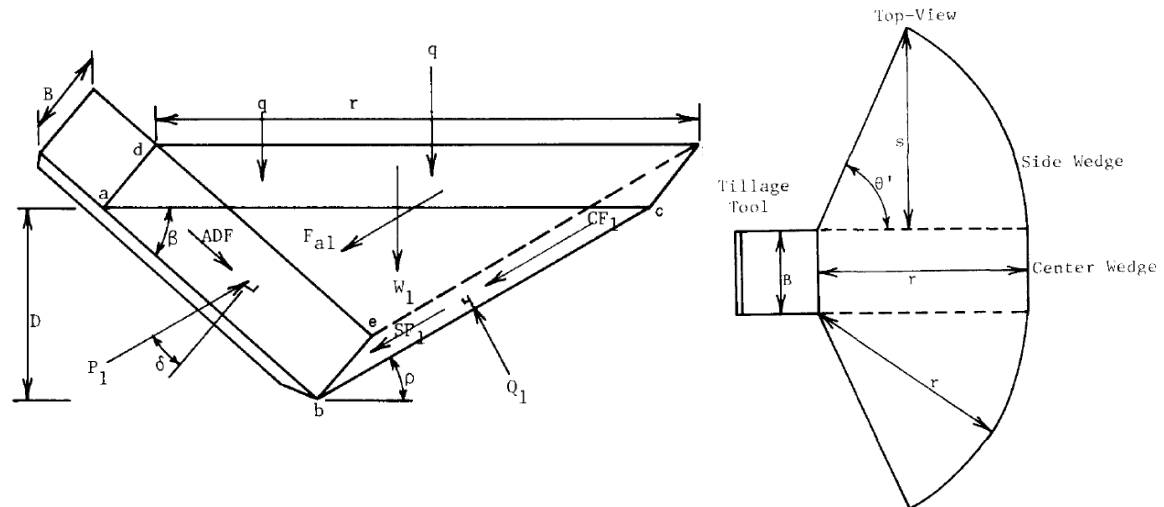


- narrow tillage tools
- includes:
  - soil-tool adhesion,
  - mass,
  - inertia,
  - Surcharge
  - cohesion terms
- same results as McKyes
- center and side wedges
- tested in soil-clay mix with 2.5, 5.1, 7.6 and 10.2-cm tool widths; of 5.1, 10.2 and 15.2-cm tool depths; of 60, 75 and 90° rake angles, and 5.4, 33.1, 67.1 and 120-cm/s tool-speeds

$$T_{sw\&p} = \left( \frac{wd}{\sin(\beta + \phi + \rho + \delta)} \right) + \left( \frac{-C_a \cos(\beta + \phi + \rho)}{\sin(\beta)} \right) + \left( \frac{g\gamma d}{2} (\cot(\beta) + \cot(\rho)) (\sin(\phi + \rho)) \right) + \left( \frac{c \cos(\phi)}{\sin(\rho)} \right) + \left( \frac{\gamma v^2 \sin(\beta) \cos(\phi)}{\sin(\beta + \rho)} \right)$$

$$H_{sw\&p} = T_{sw\&p} \sin(\beta + \delta)$$

$$V_{sw\&p} = T_{sw\&p} \cos(\beta + \delta)$$







# Zeng Model

Includes:

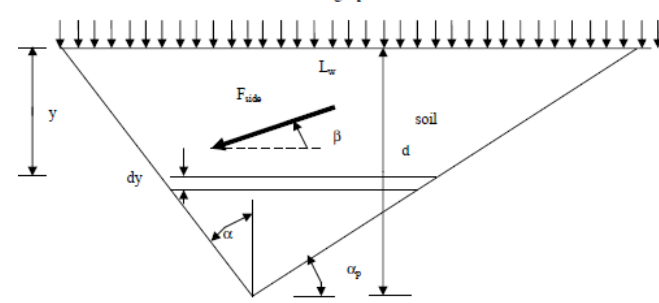
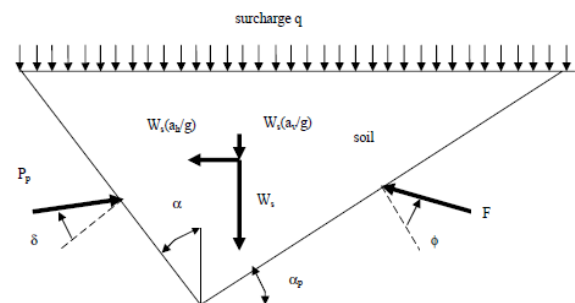
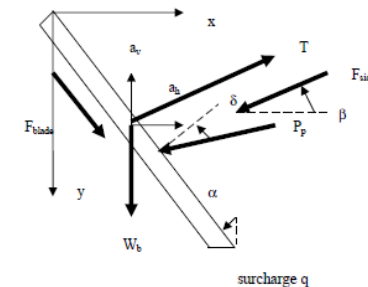
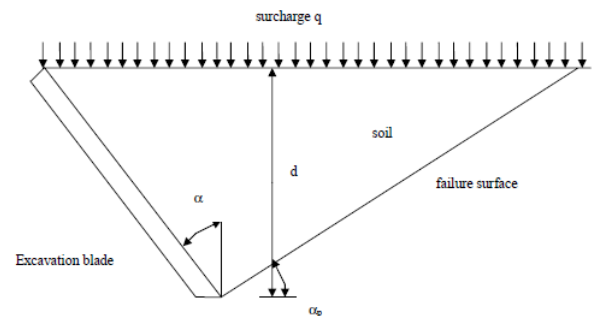
- dynamic earth pressure,
- side friction,
- surcharge,
- blade friction,
- weight of the blade,
- blade acceleration

$$T_{zeng} = \sqrt{((H_{zeng})^2 + (V_{zeng})^2)}$$

$$H_{zeng} = -wF_{blade} \sin(90 - \beta) + wP_p \cos(90 - \beta - \delta) + F_{side} \cos(\phi + \delta) + (W_b)a_h$$

$$V_{zeng} = wF_{blade} \cos(90 - \beta) + W_b + wP_p \sin(90 - \beta - \delta) + F_{side} \sin(\phi + \delta) + (W_b)a_v$$

$$P_p = 0.5K_{PE} \left(1 + \frac{a_v}{g}\right) \gamma g d^2 L + 2cdL \sqrt{K_{PE}} + K_{PE} q g d L$$



Zeng X, Burnoski L, Agui JH, Wilkinson A. Calculation of excavation force for isru on lunar surface. In: 45th AIAA Aerospace Science Meeting and Exhibit, American Institute of Astronautics and Aeronautics;2007.



# Comparison of included variables across models



Common to all models: rake angle ( $\beta$ ), soil density ( $\gamma$ ), and tool width ( $w$ )

Common except L&W: Cohesion ( $c$ ), internal friction angle ( $\phi$ ), external friction angle ( $\delta$ )

description	unit	variable	Osman	Gill & vdBerg / Blouin	swick & perumpral	McKyes	L&W sand	Balovnev bulldozer	Zeng	Qinsen & Shuren
horizontal acceleration	m/s <sup>2</sup>	$a_h$							x	
vertical acceleration	m/s <sup>2</sup>	$a_v$							x	
rake angle from horizontal	degrees	$\beta$	x	x	x	x	x	x	x	x
length of blade beneath soil surface	m	bladlength	x							
cohesion	kN/m <sup>2</sup>	$c$	x	x	x	x		x	x	x
cohesion after cutting	kN/m <sup>2</sup>	$c_0$								x
soil-tool adhesion	kN/m <sup>2</sup>	$C_a$	x		x	x				x
cutting depth	m	$d$		x	x	x	x	x	x	x
interface angle of friction	degrees	$\delta$	x	x	x	x		x	x	x
acceleration of gravity	m/s <sup>2</sup>	$g$	x	x	x	x	x	x	x	
soil unit density	kg/m <sup>3</sup>	$\gamma$	x	x	x	x	x	x	x	x
soil unit density after cutting	kg/m <sup>3</sup>	$\gamma_1$						x		x
coefficient of passive earth pressure		$K_0$							x	
tool length	m	$L$		x			x			x
friction angle after cutting	degrees	$\phi_0$								x
internal angle of friction	degrees	$\phi$	x	x	x	x		x	x	x
surcharge	kg/m <sup>2</sup>	$q$	x		x	x			x	x
blade radius of curvature	m	$R$								x
angle of failure plane from horizontal	degrees	$\rho$		x	x	x				x
part of circle, curved blade	degrees	$\theta$								x
tool height above soil surface	m	toolheight						x		
blade forward speed	m/s	$v$		x	x	x	x			
blade width	m	$w$	x	x	x	x	x	x	x	x
blade mass	kg	$W_b$							x	